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The incompatibility relation between visibility of interference and distinguishability of paths

Shogo Tanimura[1]

Department of Applied Mathematics and Physics, Graduate School of Informatics, Kyoto University, Kyoto 606-8501, Japan

Abstract

A model of the Young double-slit experiment is formulated in a fully quantum theoretical setting. The state and dynamics of a movable wall which has the double slits in it, as well as the state of a particle incoming to the double slits, are described in quantum theoretical terms. We study incompatibility between producing the interference pattern and distinguishing the particle path. It is argued that the uncertainty relation involved in the double-slit experiment is not the Ozawa-type uncertainty relation but the Kennard-type uncertainty relation of the position and the momentum of the double-slit wall. A possible experiment to test the incompatibility relation is suggested. It is also argued that various phenomena which occur at the interface of a quantum system and a classical system, including measurement, decoherence, interference and distinguishability, can be understood as different aspects of entanglement.

Keywords: double-slit experiment, uncertainty relation, Kennard-Robertson inequality, Ozawa inequality, entanglement

[1]E-mail: tanimura[AT]i.kyoto-u.ac.jp

1 Introduction

The uncertainty relation is one of the best known subjects which manifest the peculiar nature of the microscopic world. Although many people have been discussing it for a long time [1, 2, 3, 4, 5, 6, 7, 8], some confusion about the formulation and the implication of the uncertainty relation remained. Recently, Ozawa [9, 10] settled down the controversy about the uncertainty relation and established a new inequality [11] which expresses a quantitative relation between noise and disturbance in measurement. According to his formulation [11, 12, 13], a measurement process is described as an interaction process of an observed object and an observing apparatus. Suppose that the object has observables $\hat{A}$ and $\hat{B}$. The apparatus has a meter observable $\hat{M}$, which is designed to read the value of $\hat{A}$. The whole system is initialized at the time $t = 0$ and the measurement is made at a later time t. The readout of the meter is $\hat{M}(t) := e^{i\hat{H}t/\hbar}\hat{M}e^{-i\hat{H}t/\hbar}$ and the true value is $\hat{A}(0) := \hat{A}$. Their difference $\hat{N} := \hat{M}(t) - \hat{A}(0)$ is called a noise operator. The change in $\hat{B}$, $\hat{D} := \hat{B}(t) - \hat{B}(0)$, is called a disturbance operator. These expectation values

$$\varepsilon(\hat{A}) := \sqrt{\langle(\hat{M}(t) - \hat{A}(0))^2\rangle}\,, \tag{1}$$

$$\eta(\hat{B}) := \sqrt{\langle(\hat{B}(t) - \hat{B}(0))^2\rangle}\,, \tag{2}$$

$$\sigma(\hat{A}) := \sqrt{\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2}\,, \qquad \sigma(\hat{B}) := \sqrt{\langle\hat{B}^2\rangle - \langle\hat{B}\rangle^2} \tag{3}$$

are defined with respect to the initial state of the whole system. Here $\varepsilon(\hat{A})$ is the error involved in the measurement of $\hat{A}$, $\eta(\hat{B})$ is the disturbance in $\hat{B}$ caused by the measurement of $\hat{A}$, and $\sigma(\hat{A})$ is the standard deviation of $\hat{A}$ in the initial state. Heisenberg [1] concluded the inequality

$$\varepsilon(\hat{q})\,\eta(\hat{p}) \gtrsim h \tag{4}$$

for the position $\hat{q}$ and the momentum $\hat{p}$ of a particle from his famous thought experiment with a gamma-ray microscope. He stated that the microscope is an example of the destruction of the knowledge of particle's momentum by an apparatus determining its position. Von Neumann [2] constructed a model of a measurement process and proved the inequality $\varepsilon(\hat{q})\,\eta(\hat{p}) \geq \frac{1}{2}\hbar$, but his proof apparently depended on the specific model. Kennard [3] proved the inequality

$$\sigma(\hat{q})\,\sigma(\hat{p}) \geq \frac{1}{2}\hbar \tag{5}$$

in a model-independent manner. Robertson [4] generalized it to prove

$$\sigma(\hat{A})\,\sigma(\hat{B}) \geq \frac{1}{2}\Big|\langle[\hat{A}, \hat{B}]\rangle\Big| \tag{6}$$

for arbitrary observables $\hat{A}$ and $\hat{B}$. Considering their implications, we call (5) and (6) the standard-deviation uncertainty relations or the fluctuation properties intrinsic to quantum states. The Kennard-Robertson inequalities have been regarded as a mathematically rigorous formulation of the uncertainty relation but actually they do not represent the physical implication that Heisenberg originally aimed to formulate. Ozawa [11] formulated a general scheme of measurement in a manner more faithful to Heisenberg's philosophy and proved

the inequality

$$\varepsilon(\hat{A})\,\eta(\hat{B}) + \varepsilon(\hat{A})\,\sigma(\hat{B}) + \sigma(\hat{A})\,\eta(\hat{B}) \,\geq\, \frac{1}{2}\Big|\langle[\hat{A},\hat{B}]\rangle\Big|. \tag{7}$$

He also constructed concrete models [11, 13] that yield $\varepsilon(\hat{q}) = 0$ and $\eta(\hat{p}) =$ finite. His model satisfy the Ozawa inequality (7) but violate the Heisenberg inequality (4). We call (7) the noise-disturbance uncertainty relation or the indetermination involved in a measurement process.

On the other hand, the interference effect of matter wave, or the particle-wave duality of matter, is another well known peculiarity of quantum mechanics. When a beam of particles is emitted toward a wall that has double slits on it, we observe an interference pattern on a screen behind the wall. If we put some device to detect which slit each particle has passed, then the interference pattern disappears. It is impossible to distinguish the path of each particle without smearing the interference pattern. In some textbooks [14, 15] the fact that distinguishing the particle path and viewing the interference pattern are incompatible is explained as a consequence of the uncertainty relation in this way; if we knew both the position and the momentum of the double-slit wall simultaneously, we could detect which slit each particle has passed without destroying the interference pattern. However, we know that the simultaneous measurements of the position and the momentum is impossible.

Although the double-slit experiment is regarded as a pedagogical subject from the viewpoint of modern physics, it remains unclear what kind of uncertainty relation is involved there. Thus we propose a question; *Which type of the uncertainty relation, the Kennard inequality or the Ozawa inequality, prevents the simultaneous measurements of the interference and the path?* This is the question we study in this paper.

In this paper we formulate the double-slit experiment in genuine quantum theoretical terms and analyze incompatibility between distinguishing the particle path and viewing the interference pattern. Our conclusion is that the incompatibility is attributed to the Kennard-type uncertainty relation, which is the property intrinsic to the quantum state of the double-slit wall. We will propose an experiment to test this distinguishability-visibility relation.

2 Model and its analysis

Here we shall formulate a model of the Young interferometer. As shown in Fig. 1, a particle is emitted from the source, flies through the double slits on the wall, and arrives at the screen behind the wall. We call each slit as the slit 1 and the slit 2, respectively. They are separated by a distance d. The coordinate axis, which we call the x-axis, is taken to be parallel to the wall and the screen. The wall is movable along the x-axis while the screen is fixed. The coordinate and the momentum of the particle are denoted as (q,p). Similarly, the coordinate and the momentum of the double-slit wall are denoted as (Q,P). The x-coordinate of the slit 1 is $Q+\frac{d}{2}$ while the x-coordinate of the slit 2 is $Q-\frac{d}{2}$. A position eigenstate of the whole system is $|q\rangle \otimes |Q\rangle = |q,Q\rangle$. The initial state of the whole system is assumed to be

$$|\mathrm{initial}\rangle = |\psi\rangle \otimes |\xi\rangle, \tag{8}$$

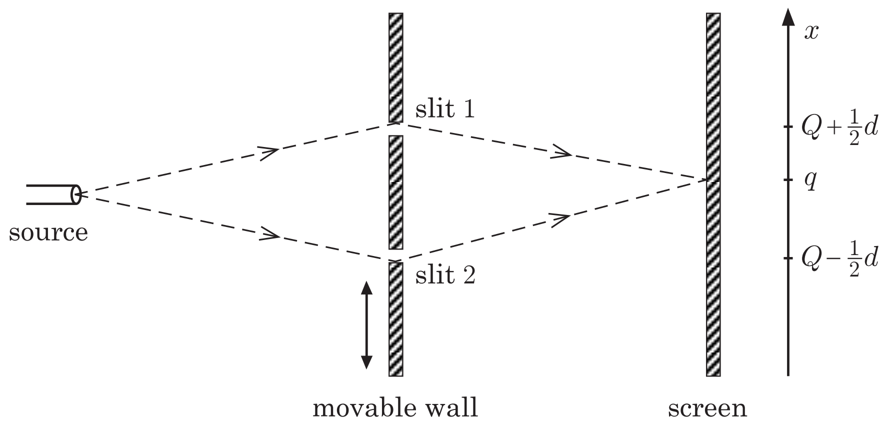

Figure 1: The Young interferometer.

which is a composite of the particle state $|\psi\rangle$ with the wall state $|\xi\rangle$. The emitted particle obeys the free-particle Hamiltonian. Its time evolution operator is

$$\hat{U}(t) = \exp\Big(-\frac{i}{\hbar}\frac{\hat{p}^2}{2m}t\Big). \tag{9}$$

We assume that the particle reaches the slits on the movable wall at the time τ.

The function of the slits are described with two projection operators, $\hat{S}_1$ and $\hat{S}_2$. If the particle arrives at the slit 1, its state becomes $\hat{S}_1|\psi\rangle$. If the particle arrives at the slit 2, its state becomes $\hat{S}_2|\psi\rangle$. They satisfy $\hat{S}_\alpha^\dagger = \hat{S}_\alpha$, $\hat{S}_\alpha^2 = \hat{S}_\alpha$, $\hat{S}_1 + \hat{S}_2 = 1$ and $\hat{S}_1\hat{S}_2 = \hat{S}_2\hat{S}_1 = 0$. We assume that when the particle hits the slit 1, it gives a momentum $+k$ to the wall. On the other hand, when the particle hits the slit 2, it gives a momentum $-k$ to the wall. Hence, after the momentum exchange, the state of the whole system becomes

$$|\text{through the slits}\rangle = e^{-ik\hat{q}/\hbar}\hat{S}_1\hat{U}(\tau)|\psi\rangle \otimes e^{ik\hat{Q}/\hbar}|\xi\rangle + e^{ik\hat{q}/\hbar}\hat{S}_2\hat{U}(\tau)|\psi\rangle \otimes e^{-ik\hat{Q}/\hbar}|\xi\rangle. \tag{10}$$

The operator $e^{-ik\hat{q}/\hbar} \otimes e^{ik\hat{Q}/\hbar}$ can be regarded as an evolution for an infinitesimal time Δt,

$$\exp\Big(-\frac{i}{\hbar}\hat{V}\Delta t\Big) = e^{ik(\hat{Q}-\hat{q})/\hbar}, \tag{11}$$

which is generated by the interaction Hamiltonian $\hat{V} = -F(\hat{Q}-\hat{q})$. Here F is a constant force and $F\Delta t = k$ is the impact. The operator $e^{ik\hat{Q}/\hbar}$ shifts the wall momentum as $e^{ik\hat{Q}/\hbar}|P\rangle = |P+k\rangle$ while the operator $e^{-ik\hat{q}/\hbar}$ shifts the particle momentum as $e^{-ik\hat{q}/\hbar}|p\rangle = |p-k\rangle$.

The particle arrives at the screen at the time $\tau+\tau'$. Then the state of the whole system becomes

$$\begin{aligned} |\text{final}\rangle &= \hat{U}(\tau')e^{-ik\hat{q}/\hbar}\hat{S}_1\hat{U}(\tau)|\psi\rangle \otimes e^{ik\hat{Q}/\hbar}|\xi\rangle + \hat{U}(\tau')e^{ik\hat{q}/\hbar}\hat{S}_2\hat{U}(\tau)|\psi\rangle \otimes e^{-ik\hat{Q}/\hbar}|\xi\rangle \\ &= |\psi_1\rangle \otimes e^{ik\hat{Q}/\hbar}|\xi\rangle + |\psi_2\rangle \otimes e^{-ik\hat{Q}/\hbar}|\xi\rangle. \end{aligned} \tag{12}$$

Here we put $\hat{U}(\tau')e^{-ik\hat{q}/\hbar}\hat{S}_\alpha\hat{U}(\tau)|\psi\rangle = |\psi_\alpha\rangle$. Then the probability for finding the particle at the position q on the screen is proportional to

$$\text{Prob}(q) \quad \propto \quad \int \Big|\langle q, Q\,|\text{final}\rangle\Big|^2 dQ$$

$$
\begin{aligned}
&= \left|\psi_1(q)\right|^2 + \left|\psi_2(q)\right|^2 + 2\,\mathrm{Re}\left\{\psi_1^*(q)\psi_2(q)\,\langle\xi|e^{-2ik\hat{Q}/\hbar}|\xi\rangle\right\} \\
&= \left|\psi_1(q)\right|^2 + \left|\psi_2(q)\right|^2 + 2\,\mathscr{V}\,\mathrm{Re}\left\{e^{i\alpha}\,\psi_1^*(q)\psi_2(q)\right\}. \qquad (13)
\end{aligned}
$$

The last term describes an interference of the two waves $\psi_1(q) = \langle q|\psi_1\rangle$ and $\psi_2(q) = \langle q|\psi_1\rangle$. The nonnegative real number $\mathscr{V}$ and the phase $e^{i\alpha}$ are defined by

$$
\mathscr{V} e^{i\alpha} = \langle\xi|e^{-2ik\hat{Q}/\hbar}|\xi\rangle. \qquad (14)
$$

The contrast of the interference fringe is proportional to $\mathscr{V}$, which is called the visibility of the interference and takes its value in the range $0 \le \mathscr{V} \le 1$. The wave function of the movable wall is denoted as $\xi(Q) = \langle Q|\xi\rangle$ and its Fourier transform is

$$
\tilde{\xi}(P) = \frac{1}{\sqrt{2\pi\hbar}} \int e^{-iPQ/\hbar}\,\xi(Q)dQ. \qquad (15)
$$

Then the visibility is expressed as

$$
\mathscr{V} = \left|\int \xi^*(Q)\,e^{-2ikQ/\hbar}\,\xi(Q)dQ\right| = \left|\int \tilde{\xi}^*(P-k)\,\tilde{\xi}(P+k)dP\right|. \qquad (16)
$$

After observing the position of the particle on the screen, we measure the momentum of the wall to detect the path of the particle. The conditional probability distribution of the momentum P is calculated from (12) as

$$
\mathrm{Prob}(P|q) \;\propto\; \left|\langle q, P\,|\mathrm{final}\rangle\right|^2 = \left|\psi_1(q)\,\tilde{\xi}(P-k) + \psi_2(q)\,\tilde{\xi}(P+k)\right|^2. \qquad (17)
$$

If the support of the initial wave function $|\tilde{\xi}(P)|$ is contained within the range $P_0 - k < P < P_0 + k$ for some P_0, then from the measured value of P we can tell the slit which the particle passed. If the measured momentum is in the range $P_0 < P < P_0 + 2k$, we can conclude that the particle hit the slit 1. On the other hand, if the measured momentum is in the range $P_0 - 2k < P < P_0$, we can conclude that the particle hit the slit 2. However, if the support of $|\tilde{\xi}(P)|$ is contained within $P_0 - k < P < P_0 + k$, the overlap integral in (16) vanishes and hence the interference fringes fade away completely.

Contrarily, if the width of the support of $\tilde{\xi}(P)$ is larger than $2k$, the visibility (16) can be nonzero. However, at that time, the probability (17) can have a nonzero interference term and hence we cannot distinguish the particle path certainly.

We summarize the above argument symbolically as

$$
\begin{aligned}
\text{Visible interference} \quad &\Leftrightarrow \quad \mathscr{V} \neq 0 \\
&\Rightarrow \quad \mathrm{supp}\,|\tilde{\xi}(P-k)| \cap \mathrm{supp}\,|\tilde{\xi}(P+k)| \;\text{ has nonzero measure.} \\
&\Leftrightarrow \quad \text{The path of the particle cannot be distinguished completely} \\
&\qquad\quad \text{by measuring the momentum of the wall.}
\end{aligned}
$$

In the above inference, the second arrow ($\Rightarrow$) cannot be replaced with the necessary and sufficient sign ($\Leftrightarrow$). For example, if we take the wave function

$$
\tilde{\xi}(P) = \begin{cases} a\,\sin\left(\frac{2\pi P}{2k}\right) & (0 \le P \le 2k) \\ b\,\sin\left(\frac{4\pi P}{2k}\right) & (-2k \le P \le 0) \\ 0 & (\text{otherwise}), \end{cases} \qquad (18)
$$

then the supports of $|\tilde{\xi}(P-k)|$ and $|\tilde{\xi}(P+k)|$ have an overlap with nonzero measure but the integral $\mathscr{V}$ vanishes.

3 Uncertainty relation

Now we discuss what kind of uncertainty relation is involved in the double-slit experiment. Let us consider the expression (16) for the visibility,

$$\mathscr{V}(k) = \left| \int e^{-2ikQ/\hbar} \, |\xi(Q)|^2 \, dQ \right|. \tag{19}$$

Suppose that the probability distribution $|\xi(Q)|^2$ has an effective width ΔQ. (A rigorous definition of ΔQ is not necessary for the following argument.) Since (19) is an oscillatory integral, to get a considerably large visibility we need to have

$$2k \, \Delta Q/\hbar \lesssim 2\pi. \tag{20}$$

On the other hand, as discussed above, to distinguish the slit which the particle passes, the initial momentum distribution of the double-slit wall should be contained within the range

$$\Delta P < 2k. \tag{21}$$

Hence, to observe a clear interference and to distinguish the path of the particle simultaneously, we need to have

$$\Delta Q \, \Delta P \lesssim 2\pi\hbar. \tag{22}$$

As a contraposition, the uncertainty relation

$$\Delta Q \, \Delta P \gtrsim 2\pi\hbar \tag{23}$$

implies that making a clear interference pattern and detecting the particle path cannot be accomplished simultaneously. This uncertainty relation (23) is a property of the initial state of the double-slit wall but it is not a relation between noise and disturbance caused by measurement. Hence, we conclude that the obstruction against the simultaneous realization of interference and path detection is the Kennard-type uncertainty relation which is intrinsic to the quantum state of the double-slit wall. This is the main result of this paper. It is to be noted that this is a conclusion of an analysis of the specific model. We do not have to take it as a universally valid statement.

On the other hand, there was a puzzle asking whether the position-momentum uncertainty relation is relevant or not to the interference-distinguishing complementarity. Storey *et al.* [16] argued that the position-momentum uncertainty relation is responsible for destroying the interference pattern. Englert *et al.* [17] took an opposite position and argued that the *which-way information* is responsible for destroying the interference pattern and that the disturbance in the momentum is not necessarily responsible. Dürr, Nonn, and Rempe [18, 19] performed experiments to prove that what destroys the interference pattern is the which-way information. Hence, Englert's argument was correct. However, what we discussed in this paper is a question asking *which type of the position-momentum uncertainty relation is responsible for destroying the interference pattern if we try to detect the path of the particle by measuring the momentum of the movable slit-wall.* And finally we concluded that the Kennard-type uncertainty relation is responsible in the context of our model.

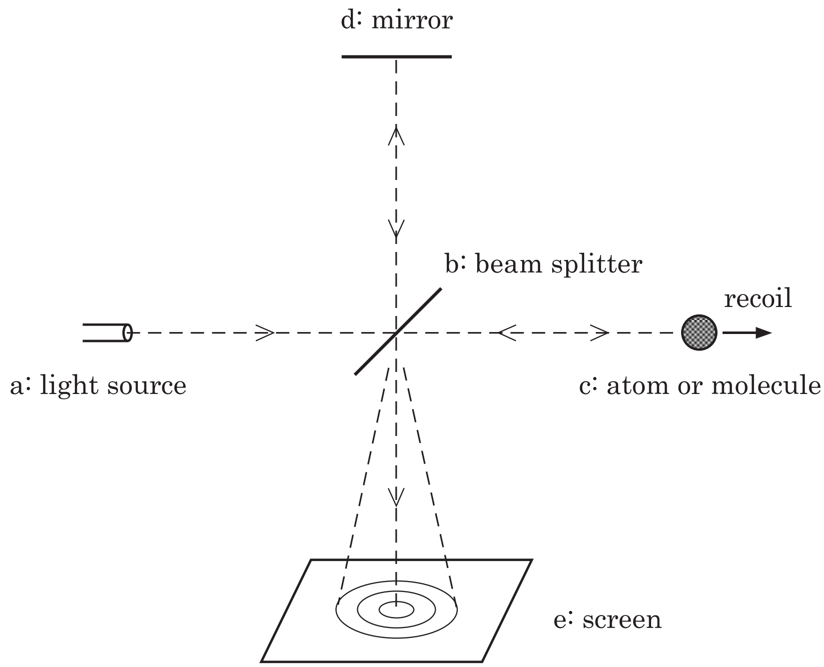

Figure 2: The Michelson interferometer.

4 Suggestions for experiments

Here we would like to suggest an experimental scheme to test the visibility relation (16). Our scheme uses the Michelson interferometer as illustrated in Fig. 2. A photon is emitted from a light source (a) and is split by a beam splitter (b) into two directions. At the end (c) of one direction an atom or a molecule is placed. The incident photon is scattered by the atom and the atom recoils. At the other end (d) a mirror is fixed. The two paths merge at the beam splitter (b) and the photon reaches the fixed screen (e). There we observe an interference pattern by accumulating photons. On the other hand, by measuring the velocity of the atom, we can infer the path of the photon; If the atom recoils out, we know that the photon took the path (c). If the atom remains stationary, we know that the photon took the path (d).

If the initial wave function of the atom is strongly localized, one will observe a clear interference pattern but fail to measure the velocity of the atom precisely. If the initial wave function of the atom has a larger spatial spread, one will measure the velocity of the atom with a smaller error but the interference pattern will become feebler. Thus the initial state of the atom determines the visibility of the interference fringe as (16). We may put a Bose-Einstein condensate (BEC) of atoms at the place (c) instead of a single atom since control and observation of the BEC are more feasible than a single atom.

We can estimate the velocity of the recoil atom. We denote the wave length of the photon as λ and the mass of the target atom as M. Then the photon momentum is $p = h/\lambda$ and the impact given to the atom is $k = 2p$. The velocity of the recoil atom is

$$v = \frac{k}{M} = \frac{2h}{M\lambda}. \tag{24}$$

Assume that the photon wave length is $\lambda = 0.5 \times 10^{-6}$ m and that we use a mercury atom as a reflector. Then the recoil velocity is $v = 7.9 \times 10^{-3}\,\mathrm{m} \cdot \mathrm{s}^{-1}$. If we use a BEC of 10^4 sodium atoms, the velocity is $v = 6.9 \times 10^{-6}\,\mathrm{m} \cdot \mathrm{s}^{-1}$. Furthermore, the argument around Eq. (20) implies that the size of the spread of the wave function of the target should be smaller than

$$\Delta Q \sim \frac{\pi\hbar}{k} = \frac{1}{4}\lambda \tag{25}$$

for making a clear interference pattern.

In the above argument we proposed a use of the Michelson interferometer. Other interference experiments, like the Hanbury-Brown-Twiss correlation [20] or the interference of photons from two light sources, which has been demonstrated by Mandel *et al.* [21], can also be modified to experiments which demonstrate the tradeoff between interference and distinguishability. It is also to be noted that Plau *et al.* [22] and Chapman *et al.* [23] had demonstrated that a change of the momentum distribution of an atom by photon emission or by photon scattering causes a change of spatial coherence of the atom. They had confirmed the Kennard-type uncertainty relation.

5 Interference and measurement as entanglements

Before closing our discussion we would like to mention that various phenomena which occur at the interface between a quantum system and a classical system, including measurement, decoherence, interference and distinguishability, can be understood on the same footing. It has been known that measurement and decoherence are formulated in the same framework using the concept of operation [12, 13, 24, 25]. The relation of the quantum theory and the classical theory is understood more suitably in terms of the micro-macro duality, which was recently proposed by Ojima [26]. Measurement processes are also considered as a kind of entanglement and understood from the viewpoint of the micro-macro duality [26, 27]. Englert [28] and Hosoya *et al.* [29] theoretically analyzed and Dürr [19] experimentally tested the complementarity between interference visibility and distinguishability. However, the formulation and analysis of interference and distinguishability, which we discussed in this paper, is still new. It would be worthwhile to mention that these superficially different phenomena, measurement, decoherence, interference and distinguishability, can be described in an unified manner.

Suppose that we have two interacting systems; one is referred as an observed object while the other is referred as an observing apparatus or an environment. The Hilbert space of the observed system is denoted as $\mathscr{H}$ and the Hilbert space of the observing system is $\mathscr{K}$. Assume that the state of the whole system is

$$c_1|\psi_1\rangle \otimes |\xi_1\rangle + c_2|\psi_2\rangle \otimes |\xi_2\rangle \tag{26}$$

with normalized vectors $|\psi_\alpha\rangle \in \mathscr{H}$ and $|\xi_\alpha\rangle \in \mathscr{K}$. Although here we consider a linear combination of only two terms, generalization to more than two terms is straightforward. When $|\psi_1\rangle$ and $|\psi_2\rangle$ are linearly independent and simultaneously $|\xi_1\rangle$ and $|\xi_2\rangle$ are also linearly independent, it is said that the two systems are entangled or that the object is in an incoherent state. Otherwise, it is said that the two systems are disentangled or that the object is in a pure state or a coherent state.

Suppose that we measure an observable $\hat{A}$ on $\mathscr{H}$. The probability that the measured value of $\hat{A}$ will be a is proportional to

$$\mathrm{Prob}_A(a) \ \propto \ \Big| c_1 \langle a|\psi_1\rangle \Big|^2 + \Big| c_2 \langle a|\psi_2\rangle \Big|^2 + 2\,\mathrm{Re}\Big\{ c_1^* c_2 \, \langle a|\psi_1\rangle^* \, \langle a|\psi_2\rangle \langle \xi_1|\xi_2\rangle \Big\}. \tag{27}$$

If $\langle \xi_1|\xi_2\rangle \neq 0$, we will observe interference between the states $|\psi_1\rangle$ and $|\psi_2\rangle$. The visibility of the interference is proportional to the absolute value of $\langle \xi_1|\xi_2\rangle$. On the other hand, if $\langle \xi_1|\xi_2\rangle = 0$, by measuring the state of the apparatus we can certainly judge the state of the object and then the interference completely disappears.

Next, suppose that we measure an observable $\hat{M}$ on $\mathscr{K}$, which may be called a meter observable. The probability that the measured value of $\hat{M}$ will be m is proportional to

$$\mathrm{Prob}_M(m) \ \propto \ \Big| c_1 \langle m|\xi_1\rangle \Big|^2 + \Big| c_2 \langle m|\xi_2\rangle \Big|^2 + 2\,\mathrm{Re}\Big\{ c_1^* c_2 \langle \psi_1|\psi_2\rangle \langle m|\xi_1\rangle^* \langle m|\xi_2\rangle \Big\}. \tag{28}$$

After the measurement the state of the object becomes

$$c_1 |\psi_1\rangle \langle m|\xi_1\rangle + c_2 |\psi_2\rangle \langle m|\xi_2\rangle. \tag{29}$$

Then the state of the observed object is disturbed by the measurement. The probability distribution of an observable $\hat{B}$ in the state (26) before the measurement is different from the one in the state (29) after the measurement. At this time the noise-disturbance uncertainty relation [11] holds.

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References

[1] W. Heisenberg, *Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik*, Z. Phys. **43**, 172 (1927); W. Heisenberg, *The Physical Principles of the Quantum Theory*, English translation by C. Eckart and F. C. Hoyt (University of Chicago Press, 1930; Dover, 1949).

[2] J. von Neumann, *Die mathematische Grundlagen der Quantenmechanik* (Springer, 1932); *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, 1996).

[3] E. H. Kennard, *Zur Quantenmechanik einfacher Bewegungstypen*, Z. Phys. **44**, 326 (1927).

[4] H. P. Robertson, *The uncertainty principle*, Phys. Rev. **34**, 163 (1929).

[5] V. B. Braginskii and Yu. I. Vorontsov, *Quantum-mechanical limitations in macroscopic experiments and modern experimental technique*, Soviet Physics Usp. **17**, 644 (1975).

[6] C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmermann, *On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle*, Rev. Mod. Phys. **52**, 341 (1980).

[7] H. P. Yuen, *Contractive states and the standard quantum limit for monitoring free-mass positions*, Phys. Rev. **51**, 719 (1983).

[8] C. M. Caves, *Defense of the standard quantum limit for free-mass position*, Phys. Rev. Lett. **54**, 2465 (1985).

[9] M. Ozawa, *Measurement breaking the standard quantum limit for free-mass position*, Phys. Rev. Lett. **60**, 385 (1988).

[10] J. Maddox, *Beating the quantum limits*, Nature **331**, 559 (1988).

[11] M. Ozawa, *Physical content of Heisenberg's uncertainty relation: limitation and reformulation*, Phys. Lett. A **318**, 21 (2003).

[12] M. Ozawa, *Quantum measuring processes of continuous observables*, J. Math. Phys. **25**, 79 (1984).

[13] M. Ozawa, *Uncertainty relations for noise and disturbance in generalized quantum measurements*, Ann. Phys. **311**, 350 (2004).

[14] A. Messiah, *Quantum Mechanics*, translated from French by G. M. Temmer, Chap. 4, Sec. 18 (North-Holland, 1961; Dover, 1999).

[15] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Vol. 3, Secs. 1-8, 3-1, 3-2 (Addison-Wesley, 1966).

[16] P. Storey, S. Tan, M. Collett, and D. Walls, *Path detection and the uncertainty principle*, Nature **367**, 626 (1994).

[17] B.-G. Englert, M. O. Scully, and H. Walther, *Complementarity and uncertainty*, Nature **375**, 367 (1995). P. Storey, S. Tan, M. Collett, and D. Walls, Nature **375**, 368 (1995).

[18] S. Dürr, T. Nonn, and G. Rempe, *Origin of quantum-mechanical complementarity probed by a 'which-way' experiment in an atom interferometer*, Nature **395**, 33 (1998).

[19] S. Dürr, T. Nonn, and G. Rempe, *Fringe visibility and which-way information in an atom interferometer*, Phys. Rev. Lett. **81**, 5705 (1998).

[20] R. Hanbury Brown and R. Q. Twiss, *Correlation between photons in two coherent beams of light*, Nature **177**, 27 (1956); *A test of a new type of stellar interferometer on Sirius*, Nature **178**, 1046 (1956).

[21] G. Magyar and L. Mandel, *Interference fringes produced by superposition of two independent maser light beams*, Nature **198**, 255 (1963); L. Mandel, *Quantum effects in one-photon and two-photon interference*, Rev. Mod. Phys. **71**, S274 (1999).

[22] T. Pfau, S. Spälter, Ch. Kurtsiefer, C. R. Ekstrom, and J. Mlynek, *Loss of spatial coherence by a single spontaneous emission*, Phys. Rev. Lett. **73**, 1223 (1994).

[23] M. S. Chapman, T. D. Hammond, A. Lenef, J. Schmiedmayer, R. A. Rubenstein, E. Smith, and D. E. Pritchard, *Photon scattering from atoms in an atom interferometer: Coherence lost and regained*, Phys. Rev. Lett. **75**, 3783 (1995).

[24] K.-E. Hellwig and K. Kraus, *Pure operations and measurements*, Commun. Math. Phys. **11**, 214 (1969).

[25] E. B. Davies and J. T. Lewis, *An operational approach to quantum probability*, Commun. Math. Phys. **17**, 239 (1970).

[26] I. Ojima, *Micro-Macro Duality in Quantum Physics*, pp.143-161 in Proc. Int'l Conf. on Stochastic Analysis, Classical and Quantum (World Scientific, 2005), e-print arXiv: math-ph/0502038.

[27] I. Ojima and M. Takeori, *How to observe quantum fields and recover them from observational data? —Takesaki duality as a Micro-Macro duality—*, Open Sys. & Inf. Dyn. **14**, 307 (2007), e-print arXiv: math-ph/0604054.

[28] B.-G. Englert, *Fringe visibility and which-way information: an inequality*, Phys. Rev. Lett. **77**, 2154 (1996).

[29] A. Hosoya, A. Carlini, and S. Okano, *Complementarity of entanglement and interference*, Int'l J. Mod. Phys. C **17**, 493 (2006).

[30] I. Ojima, an unpublished note on an algebraic formulation of interference phenomena (dated 21 March 2002), which was brought to my attention via a private communication with him (2007).